The EPR Experiment

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Abstract

In this article, the Einstein-Podolsky-Rosen(EPR) experiment will be the basis for a philosophical discussion on its the implication on physical reality. The EPR's claim was that quantum theory was incomplete by introducing local hidden variables and formulating their argument around the assumption of reality and locality. I will derive the Bell inequality, and show how the EPR's claim fell through in light of Alain Aspect's experimental verification in favour of the quantum mechanics prediction. This shows that either one or both of the EPR assumption must be inappropriate. I will then argue why the classical concept of reality must be given up to preserve the locality assumption by raising certain philosophical implication of causality if locality is given up instead. The relinquishing of reality is justified in that our perception of reality is prejudiced towards macroscopic entities such that we find the quantum description, though a correct description of reality, implausible. However, it can be accepted, provided we depart from our classical definition of physical reality.

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1 Introduction

Quantum theory, with all its somewhat counter-intuitive and bizarre predictions and phenomena, is without doubt a difficult concept to comprehend in its entirety. First and foremost, it challenges our conventional concept of reality. One of the most very basic phenomenon of quantum mechanics says that we can never have a precise measurement of *both* the position *and* the momentum of a particle at the same time according to Heisenberg's uncertainty principle. So what happens when we try to measure the position of a particle? Born's statistical interpretation says that the probability of finding the particle at a particular point is given by the square of the wavefunction [1]. Thus quantum mechanics is very much a statistical theory. And this introduces a kind of indeterminacy into quantum mechanics and raises the question of where the particle was just before the measurement.

There are three main schools of thought regarding this quantum indeterminacy. The **realist** position says that the particle was really at a certain position, but that quantum mechanics was unable to tell us so and hence incomplete. Thus they, in particular Einstein, advocated that there must be some hidden variables to supplement quantum mechanics. The **orthodox** position says that the particle wasn't really anywhere and that the act of measurement forced the particle to take a stand. This is also known as the Copenhagen interpretation associated with Bohr. The third position is the **agnostic** position, which refuses to answer the question, claiming that there is no sense in making assertions about a particle before the measurement.

Einstein, a staunch realist, could not come to terms with the orthodox interpretation, thus formulated the EPR thought experiment, to try and show that quantum mechanics is incomplete. The EPR experiment, though simple, raises numerous important fundamental ontological problems of physical theory, which I will move on to discuss in this paper. But before we move on, it is of paramount importance to set out clearly the definition of certain key terms that will be used throughout the paper.

Physical Reality - If without in any way disturbing a system, we can predict with probability equal to unity the value of a physical quantity, then there exists an element of physical reality corresponding to the physical quantity [4].

Completeness - A theory is complete if every element of reality has a counterpart in it [4].

Correctness - A theory is totally correct if every element of the theory has a counterpart in reality [4].

Locality - Elements of reality pertaining to one system cannot be effected by measurements performed *at a distance* on another system. This principle assumes that anything that happen at a given location has only local effects i.e. two systems that are spatially separated with no interaction cannot influence one another [4].

Determinism - Sufficient information at t_o allows prediction of a specific result at a later time t (Cushing, [5]).

Causality - A specific preceding event (or cause) for every effect (Cushing, [5]).

Separability - Spatially separated systems always have independently definable properties and existence (Cushing, [5]).

2 The EPR Experiment

The EPR paper was first jointly published in 1935 by Einstein, Podolsky and Rosen. The paper describes a thought experiment designed to prove that the realist position is the only sustainable one on purely theoretical grounds [1], using the reality and locality conditions. Their claim was that quantum theory was incomplete and not that it is incorrect. Bohm later modified the experiment into a simpler and clearer form and it is this that will be described here, following [1] and [4] closely.

We first consider the decay of a neutral pi meson into an electron and a positron. Assuming that the pi meson is initially at rest, the electron and positron will fly off in opposite direction. Assuming also that the pi meson have zero spin, and by the conservation of angular momentum, the electron and positron must have opposite spins and they are in the following singlet state:

$$\frac{1}{\sqrt{2}}(\uparrow_{-}\downarrow_{+} + \downarrow_{-}\uparrow_{+}). \tag{1}$$

We let the particles fly very far apart, and make a measurement on the electron at time t_1, and let's say that it turns up with a spin up. We can then say with probability equal to unity that the spin of the positron at any time $t_2 > t_1$ must be down. We thus infer from the reality condition that there exists for any time $t_2 > t_1$ an element of reality associated with spin of the positron. We then infer from the locality condition that the measurement of the electron have no effect on the outcome of measurement of the positron. Thus, there must also be an element of reality associated with the spin of the positron at a time $t_o < t_1$. However, at time t_o before any measurements were made, the state of the system was given by Eqn 1. Therefore, we encounter an element of reality (the spin down of the positron) that has no counterpart in the quantum theory, and hence conclude from the completeness condition that quantum theory must be incomplete. They proposed the (local) hidden variable theory, which basically claims that there are certain yet unknown variables governing the behaviour of the particles in addition to quantum theory.

This, in essence, was line of attack of the EPR experiment. Advocates of the orthodox interpretation have raised objections to the argument, but no conclusive proof was produced in favour of either side until Bell derived his famous inequality that provided a basis for experiments to be conducted to test the hypothesis.

3 Bell's Theorem

In 1964, J. S. Bell derived a set of inequality which is able proved conclusively that *any* local variable theory is *incompatible* with quantum mechanics. A detailed derivation of the inequality is given here, following [1] closely.

The hidden variable theory proposed that there is some other quantity, λ, needed in addition to the wavefunction, Ψ, to characterize the state of a system fully. At this stage, we have no idea what function λ takes, or how to measure or calculate it. [1] In Bell's paper, he suggested a generalization of the EPR/Bohm experiment. Instead of orienting the electron and positron detectors along the same direction, he allowed them to rotate independently. The first measures the component of the electron spin in the direction of the unit vector $\mathbf{a}$, and the second measures the component of the positron spin in the direction of the positron spin in the direction $\mathbf{b}$. (See Fig 1)[1]

The spins of the electron and positron is recorded in units of $\hbar/2$ and each detector registers either $+1$(for spin up) or -1(for spin down) along

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[1] Figure adapted from [1]

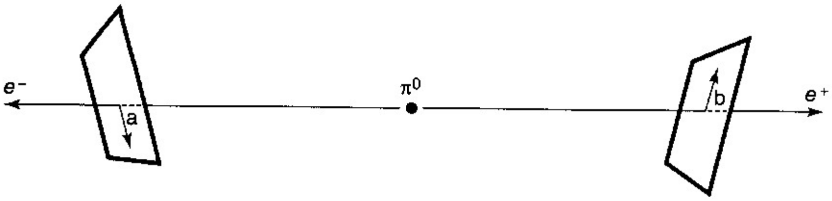

Figure 1: Bell's version of the EPR-Bohm experiment: detectors independently oriented in directions **a** and **b**.

the direction in question. Thus the result for each run of the experiment is recorded and tabulated. An typical example is shown:

electron	positron	product
$+1$	-1	-1
$+1$	$+1$	$+1$
-1	$+1$	-1
$+1$	-1	-1
-1	-1	$+1$
$\vdots$	$\vdots$	$\vdots$

Bell then went on to calculate the average value of the product of the spins, for a given set of detector orientations. We call this average $P(\mathbf{a}, \mathbf{b})$. Thus if the detectors are parallel (**b**=**a**), we recover the original EPR-Bohm configuration. In this case, the product is always -1 since one is spin up, the other spin down. Hence, the average is

$$P(\mathbf{a}, \mathbf{a}) = -1 \tag{2}$$

Also, if they are anti-parallel ($\mathbf{b}=-\mathbf{a}$), then every product is $+1$, so

$$P(\mathbf{a}, -\mathbf{a}) = +1 \tag{3}$$

Therefore, for arbitrary orientations, quantum mechanics predicts

$$P(\mathbf{a}, \mathbf{b}) = -\mathbf{a} \cdot \mathbf{b} \tag{4}$$

Bell went on to show that this result is impossible in any local hidden variable theory. The argument supposes that the "complete" state of the electron/positron system is characterized by the hidden variables λ. λ varies in some way that we cannot yet control or understand. The locality assumption says that anything that happens at a given location has only local effect, i.e. two systems that are spatially separated with no interaction cannot influence each other. Hence, the outcome of the electron measurement is independent of the orientation ($\mathbf{b}$) of the positron detector, which may be chosen by the experimenter at the positron end just before the electron measurement is made, and hence too late for subluminal message to get back to the electron detector. Thus, there exists some function $A(\mathbf{a}, \lambda)$ which gives the result of an electron measurement, and some other function $B(\mathbf{b}, \lambda)$ for the positron measurement. These functions can only take on the values ± 1:

$$A(\mathbf{a}, \lambda) = \pm 1; \qquad B(\mathbf{b}, \lambda) = \pm 1. \tag{5}$$

Therefore, for example, when the detectors are aligned, the results are perfectly anti-correlated:

$$A(\mathbf{a}, \lambda) = -B(\mathbf{a}, \lambda), \tag{6}$$

The average of the product of the measurements is then given by

$$P(\mathbf{a}, \mathbf{b}) = \int \rho(\lambda) A(\mathbf{a}, \lambda) B(\mathbf{b}, \lambda) d\lambda, \tag{7}$$

where $\rho(\lambda)$ is the probability density for the hidden variable. The behaviour of $\rho(\lambda)$ must be nonnegative and must satisfy the normalization condition $\int \rho(\lambda) d\lambda = 1$, just like any other probability density. Using Eqn (6), we can eliminate B:

$$P(\mathbf{a}, \mathbf{b}) = -\int \rho(\lambda) A(\mathbf{a}, \lambda) A(\mathbf{b}, \lambda) d\lambda. \tag{8}$$

We then introduce a third arbitrary unit vector $\mathbf{c}$, then

$$P(\mathbf{a}, \mathbf{b}) - P(\mathbf{a}, \mathbf{c}) = -\int \rho(\lambda) \left[A(\mathbf{a}, \lambda) A(\mathbf{b}, \lambda) - A(\mathbf{a}, \lambda) A(\mathbf{c}, \lambda) \right] d\lambda. \tag{9}$$

Since $[A(\mathbf{b},\lambda)]^2 = 1$, we can write:

$$P(\mathbf{a},\mathbf{b}) - P(\mathbf{a},\mathbf{c}) = -\int \rho(\lambda)\left[1 - A(\mathbf{b},\lambda)A(\mathbf{c},\lambda)\right] A(\mathbf{a},\lambda)A(\mathbf{b},\lambda)d\lambda. \tag{10}$$

But it follows from Eqn 5 that

$$-1 \leq [A(\mathbf{a},\lambda)A(\mathbf{b},\lambda)] \leq \pm 1, \tag{11}$$

and

$$\rho(\lambda)[1 - A(\mathbf{b},\lambda)A(\mathbf{c},\lambda)] \geq 0. \tag{12}$$

Therefore, we can conclude that

$$|P(\mathbf{a},\mathbf{b}) - P(\mathbf{a},\mathbf{c})| \leq -\int \rho(\lambda)\left[1 - A(\mathbf{b},\lambda)A(\mathbf{c},\lambda)\right] d\lambda, \tag{13}$$

or, more simply,

$$|P(\mathbf{a},\mathbf{b}) - P(\mathbf{a},\mathbf{c})| \leq 1 + P(\mathbf{b},\mathbf{c}). \tag{14}$$

This is the famous **Bell inequality**. This holds for *any* local hidden variable theory, for we have not made any assumptions as to the nature or number of the hidden variables or their probability density distribution. However, a close comparison of the quantum mechanical prediction (Eqn 4) and the Bell's inequality shows that they are incompatible.

For example, suppose all three vectors lie in the same plane, with $\mathbf{c}$ making an angle $\pi/4$ with $\mathbf{a}$ and $\mathbf{b}$. In this case, quantum mechanics says (Eqn 4)

$$P(\mathbf{a},\mathbf{b}) = 0, \qquad P(\mathbf{a},\mathbf{c}) = P(\mathbf{b},\mathbf{c}) = -0.707, \tag{15}$$

which is patently inconsistent with Bell's inequality, where we have:

$$\begin{aligned} |P(\mathbf{a},\mathbf{b}) - P(\mathbf{a},\mathbf{c})| &= 0.707, \qquad (16)\\ 1 + P(\mathbf{b},\mathbf{c}) &= 1 - 0.707 = 0.293. \end{aligned}$$

This clearly violates Bell's inequality. Therefore, if Einstein was right, then, not only is quantum mechanics incomplete, it is totally wrong. But on the other hand, if quantum mechanics predictions are right, then no local hidden variable theory is ever admissible to supplement quantum mechanics. Either the notion of locality or that of reality must be given up. This indeed is the crucial aspect of Bell's theorem as it provides us with a means to decide if quantum mechanics is really incomplete. Many experiments have been carried out in light of this discovery, the most conclusive of which is the Aspect experiment, which is the subject of the next section.

4 The Aspect Experiment

In 1982, Alain Aspect and his colleagues conducted an experiment to test the Bell's inequality at the Institute of Optics at the University of Paris at Orsay. [2] The experimental setup involve the source, which will emit two photons in opposite directions to the two detectors on either end of the setup. The photons were originally in the singlet state, and by the conservation of angular spin, this implies that they must have opposite spins when they are separated. (See Fig 2)[2]

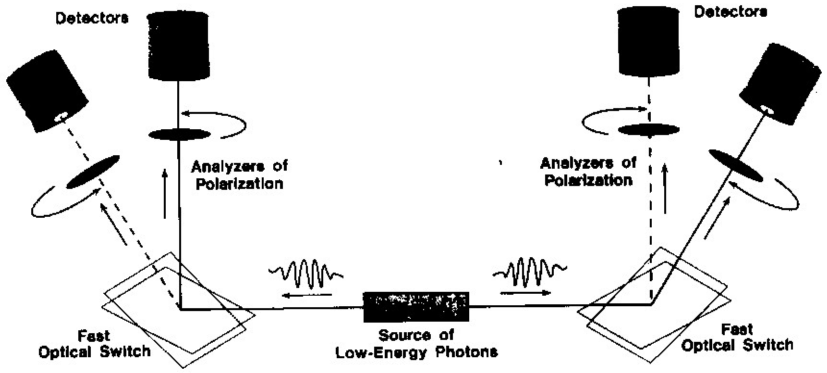

Figure 2: Experimental setup of the Aspect's experiment

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[2] Figure adapted from [2]

The two detectors on each end of the beam are set to different analyzer settings such that the possible combination of outcomes can be calculated and compared to Bell's inequality. The details of the settings are of a technical nature and the exact calculation of the probabilities is beyond the scope of this paper. Suffice it to say that the experimental data collected can be complied using classical probability theories to give the required probability to test the Bell's inequality.

One crucial feature of the experiment is that the choice between the orientations of the polarization analyzers is made by optical switches while the photons are flying away from each other. This will be able to test the locality assumption. If locality holds, i.e. if no signal can travel faster than light, the setting of the analyzers on one side cannot be communicated to or have an effect on the photon on the other side. This is ensured by the optical switch, which changes its setting while the photon is in flight, alternating the beam between the two analyzers every 10^{-8} seconds[3]. And with a distance of 12 meters between them, no signal traveling at the speed of light could be presumed to carry information between the filters. Hence, we have two detectors on each end with rapidly alternating analyzer settings, well separated in space, and the decision of the setting being made at the last possible moment.

Therefore, we can see that there is no obvious way how the result on the right-hand side could depend on which detector the left-hand photon is sent to. Since no such dependence exists, then Bell's inequality cannot be violated by virtue of the locality assumption. However, experiments carried out and repeated over again showed that quantum mechanical predictions were confirmed and that Bell's inequality was violated. This goes to prove that either the locality assumption or the reality condition imposed made in the first place must be erroneous.

5 Philosophical Discussion

5.1 Quantum theory and physical reality

From the discussion above, we note that there has never been any mention of rejecting quantum theory and proposing a new theory to describe the physics of microscopic scale. This is because the probabilistic nature of quantum theory has been so tested to give consistent result that agree with measurements that it has gained a firm status in physics. It serves well as a means of doing physics in a practical sense, but by itself, makes no claim to giving an ontological understanding of physical reality. It fails in providing us with a *picture* of reality. Folse, referring to Bohr, asserted that, "Whatever pictures we conceive of the spatio-temporal evolution of the independently existing 'isolated' system is an 'abstraction' or 'idealization' in Bohr's language and not a conceptual rendering of the characteristics of an independent physical reality." (Folse, [5]) Therefore, to assign a physical reality to the 'isolated' systems (perhaps an ensemble of particles) in question prior to a measurement is ascribing to quantum theory an issue it was never meant to address. This is central in the formulation of quantum mechanics as the properties of the system are "definable and observable only through their interaction with other systems." (Folse, quoting Bohr, [5]) In essence, these "independently existing 'isolated' system", being an abstraction, should only be viewed as being "symbolic" rather than "representational".

Being symbolic does not mean that there's no physical reality attached to it. On the contrary, the probability for obtaining a particular outcome after making a measurement is definitely a "real" quantity and may be "empirically determined ... if I create the same ψ-function very often" (Folse, [5]) and make the same measurement each time. It is the single measured value of a particular measurement that is problematic. We cannot say anything more on the system prior to measurement. It then leads to a problem of interpretating what actually happens at the act of measurement. Many interpretation theories have been raised such as the "many worlds" interpretation amongst others, which will not be discussed here. Suffice it to say that the theories, though providing an means of conceptualizing the situation, lack indication that it is the "correct" description of physical reality.

So what should we make of the reality aspect of quantum mechanics? Bohr argues that "one should not 'throw out reality'; instead in effect, he puts forward an argument for the need for a new conception of physical reality." (Folse, [5]) This calls for a radical departure from our familiar classical concepts of physical reality. Physical reality will have to be redefined. In effect, quantum theory still describes physical reality and it will only be coherent in its new redefined status, but not in the basis of classical concepts.

The question that remains now is whether quantum theory is complete. From the EPR experiment described above, we come to the conclusion that either the locality assumption or physical reality has to be reconceptualized in view of the correct and consistent prediction of quantum mechanics.

5.2 Why locality must be preserved

The principle of locality has its roots in relativity theory. One of the postulates of special relativity is that no signal can travel faster than the speed of light. This sets an upper bound on the speed of *all* physical bodies or signals in the universe and means that it takes finite time for causal influences to be effected. This essentially rule out any "spooky action-at-a-distance" that Einstein disliked so much. Or in a more precise technical terminology, spacelike separated events cannot causally influence each other. And in the context of the EPR experiment, this means that the act of measurement at one end of the beam, being spacelike separated from the other end, cannot *causally influence* the outcome of the measurement on the other end of the beam. This is what locality asserts, which i strongly agree.

Given the numerous successful predictions of relativity theory such as the phenomena of the bending of light and gravitational redshift, it is unlikely that relativity will fail. However, it is not so much that relativity theory cannot accommodate superluminal travel, but rather the logical implications that prohibits superluminal causal influences. In fact, it is easy to incorporate particles that travel at superluminal speed in relativity theory, such as the postulated tachyon. It is the implications that arise from such superluminal causal influences that's problematic. For example, the tachyon is easily shown to be traveling backwards in time in certain inertial frames of reference. To push the example to the extreme, you can in theory go back in time and kill your own grandfather, which will lead to serious logical contradictions.

We can then quite confidently disqualify any superlumial causal influences, thereby preserving the principle of locality. However, the fact that a spin up measurement on one end implies that a spin down will be observed at the other end clearly indicates that the two particles must be intrinsically linked. We have already ruled out that the measurement at one end of the beam cannot *causally influence* the outcome of the measurement on the other end of the beam. So how do we address this apparent contradiction? Surely, the measurement at one end of the beam has influenced the other particle somehow, if not how did the second particle know what spin state to assume? The crucial phrase here is "causal influence". Note that though the measurement at one end cannot *causally influence* the other particle, but that it can still influence in another sense. This second type of influence is essentially the principle of separability defined above and applied to this case, asserts that quantum theory is in essence nonseparable.

5.3 Nonseparability of quantum mechanics

The nonseparability of quantum mechanics says that spatially separated systems does not have independently definable properties and existence. But what does this nonseparability of quantum mechanics mean? It means that it is meaningless to consider, in the context of the EPR experiment, the state of a single particle alone. The two particles is in a so-called entangled state. They are intrinsically linked to each other. To push it to an extreme, all particles will then be intrinsically linked to one another, as all particles originated from the Big Bang itself.

This notion is the starting point for the concept of holism, on which Stenger puts forward his argument of a holistic worldview. [6] He argues that the universe should be taken as a whole since every particle is intrinsically linked, all originating from the Big Bang itself. This notion is interesting and is related to the new age eastern metaphysical ideas, but is however, beyond the scope of the present paper.

The nonseparability of quantum mechanics is a new facet of physical reality that we have to come to terms with. And it is exactly this redefinition that we have to incorporate in our new definition of physical reality. However, it is nevertheless difficult to comprehend or even visualize this scenario. An apt analogy is when Galileo proposed that the planets revolve around the sun, as opposed to earth being at the center of the universe. This is an extremely radical view, which turns out to be correct. This constitutes a paradigm shift, where familiar ideas have to be reconceptualized. Thus we have to remind ourselves that, although the classical viewpoints is familiar and have been tested rigorously to be successful, it could be subjected to radical changes as well. We might after all be facing another paradigm shift here.

Therefore the argument here is that the nonseparable quantum theory has proved to be correct so far. We should not reject it on grounds of conflict with conventional physical reality. Instead, having argued above that locality must be preserved, it is only left with this redefinition of physical reality, that can reconcile the contradiction raised by the EPR experiment.

6 Afterword

Having discussed the general theoretical arguments for a nonseparable quantum theory, whilst preserving locality, it is clear what step we should take next. It is confronting this new facet of physical reality and perhaps developing and modifying the theory into a more representational form. The argument in this paper, of course, lacks logical and technical rigour to quality as a conclusive argument. However, suffice it to say that it serves to provide a general framework for an attempt at a comprehensive argument for nonseparbility. Hence, it is inevitable that we give up our cherished concept of classical physical reality, given that locality must be preserved.

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